ON QUANTUM OPTIMAL TRANSPORT

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Abstract: We analyze a quantum version of the Monge–Kantorovich optimal transport problem. The quantum transport cost related to a Hermitian cost matrix C is minimized over the set of all bipartite coupling states ρ^{AB} with fixed reduced density matrices ρ^A and ρ^B of size m and n. The minimum quantum optimal transport cost $T_C^Q(\rho^A, \rho^B)$ can be efficiently computed using semidefinite programming. In the case $m = n$ the cost T_C^Q gives a semidistance if and only if C is positive semidefinite and vanishes exactly on the subspace of symmetric matrices. Furthermore, if C satisfies the above conditions, then $\sqrt{T_C^Q}$ induces the quantum Wasserstein-2 distance. Taking the quantum cost matrix C^Q to be the projector on the antisymmetric subspace, we provide a semi-analytic expression for $T_{C^Q}^Q$ for any pair of single-qubit states and show that its square root yields a transport distance on the Bloch ball. Numerical simulations suggest that this property holds also in higher dimensions. Assuming that the cost matrix suffers from decoherence and that the density matrices are diagonal, we study the quantum-to-classical transition of the Earth mover's distance, propose a continuous family of interpolating distances, and demonstrate that the quantum transport is cheaper than the classical one. Furthermore, we introduce a related quantity — the SWAP-fidelity — and compare its properties with the standard Uhlmann–Jozsa fidelity. We also discuss the quantum optimal transport for general d-partite systems.

Keywords: Quantum optimal transport, classical optimal transport, semidefinite programming.

References

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